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Optimum Fiber Orientation Angle of Multiaxially Laminated Composites Based on Reliability

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Abstract

THIS paper deals with the optimum design of multiaxially laminated fibrous composites under probabilistic conditions of loads and material properties. The first-ply-failure criterion is adopted, and the reliability analysis of biaxial, triaxial, and tetra-axial laminates under the probabilistic in-plane stresses is conducted by a method based on the advanced first-order second-moment method. The optimum fiber orientation angles of the multiaxial laminates are determined based on reliability. The optimum fiber orientation angles are found to be much different between the deterministic and the probabilistic conditions, and the reliability of the multiaxial laminates increases with the increase in the number of orientation axes under the probabilistic conditions, that is, the optimum laminates approach to a quasi-isotropic configuration.

Contents

Many investigations have been carried out on the determination of the fiber orientation angles, but most of them yield optimum laminate configurations for deterministic conditions where the strengths and the loads are assumed to have no variations. Miki et al.[1,2] developed a method to evaluate the reliability of unidirectional fibrous composites based on the advanced first-order second-moment (AFOSM) method and found that the optimum fiber angle that yields the maximum reliability changed with the increase in the variation of the applied load. In this paper, a multiaxial laminated composite is treated as a structural system, with each ply considered as one element and with the fiber orientation angles as shape design variables.

A symmetric laminated plate composed of N plies is subjected to in-plane stresses S_1, S_2, and S_6 where subscripts 1, 2, and 6 represent the major plate axis, the axis perpendicular to the 1 axis, and shear with respect to the 1-2 axes, respectively. Each ply has its own fiber orientation angle. The ply stresses are calculated by using the classical laminate theory.

The Tsai-Wu criterion is used as a limit state function M of each ply in multiaxial laminates. The limit state function indicates that $M_i \leq 0$ is the failure of the ith ply, whereas $M_i > 0$ is the nonfailure of the ith ply. The applied stresses and the strengths of each ply are treated as basic random variables X. The first-ply-failure criterion is adopted, that is, the failure event of a laminate becomes the union of failure events for all of the plies. The base random vector X is transformed into the independent standard normal vector U, and the resulting limit state functions in U space are expressed as $h_i(U)$.[3-5]

As illustrated in Fig. 1, the boundary between the safety region and the failure region for a laminate consists of multiple nonlinear limit state functions. A new algorithm is developed to select multiple appropriate points on the boundary. Based on those points, the multiple-checkpoint method[6] is applied to evaluate the system reliability of a laminate.

The limit state functions $h_i(U)$ are approximated by the linearized limit state functions with respect to the selected points $m_i(U)$, and the failure probability is represented as follows:

$$P_f = P\left[\bigcup_{i=1}^{N_B} m_i(U) \leq 0\right] \tag{1}$$

Equation (1) is estimated by evaluating its upper or lower bound, e.g., Ditlevsen's bound.[3,4] A generalized reliability index[4] is defined by using the estimated failure probability and the standard normal probability distribution function, i.e., $\beta = \Phi^{-1}(1 - P_f)$. The reliability-based optimum design problem for multiaxial laminates is to find the optimum fiber orientation angles that give the maximum system reliability of the laminate, i.e.,

$$\text{Find } \theta_i \quad (i = 1, 2, \ldots, N) \quad \text{such that} \quad \beta \rightarrow \text{maximum} \tag{2}$$

Biaxial laminates $[+\theta/-\theta]_s$, triaxial laminates $[0\text{ deg}/+\theta/-\theta]_s$, and tetra-axial laminates $[0\text{ deg}/+\theta/-\theta/90\text{ deg}]_s$ are considered. Orientation angle θ is the design variable. The plies having the same fiber angle are assumed to fail simultaneously. The ply ratios for the same orientation angles are given as constants or are optimized along with the orientation angles.

The material used for the calculations is a typical graphite/epoxy. The coefficients of variation of the strengths and the standard deviations of the applied stresses are assumed to be constant. All of the strengths and the applied stresses are assumed to be uncorrelated normal variables.

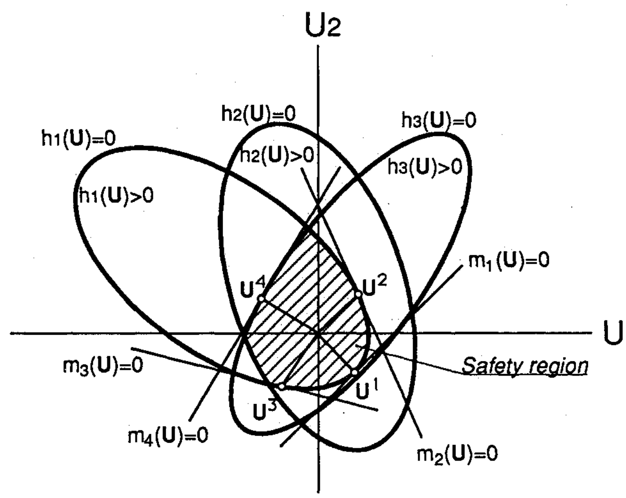

Fig. 1 Multiple-checkpoint method for evaluating reliability.

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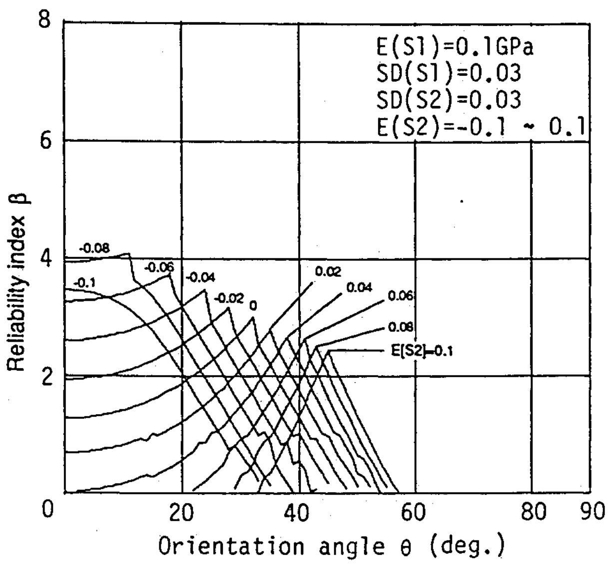

Fig. 2 Reliability of biaxial laminates for $E(S_2) = -0.1 \sim 0.1$.

Figure 2, in which $E(.)$ and $SD(.)$ represent the mean value and the standard deviation, respectively, shows the reliability indices for biaxial laminates under the probabilistic in-plane stress conditions. The optimum fiber orientation angle yielding the maximum reliability varies from 0 to 45 deg when the stress along the 2 axis changes from compression to tension, with the stress along the 1 axis being kept in tension. The reliability of the triaxial and tetra-axial laminates as a function of the orientation angle is evaluated similarly. The reliability of the biaxial laminate is found to be much lower than that of the other two types of laminates, and the reliability of the tetra-axial laminate is higher than that of the triaxial laminate when they are subjected to a compressive stress along the 2 axis.

These results are much different from the deterministic result, where the biaxial laminate gives the highest strength. This stems from the high sensitivity of the strength ratio of the biaxial laminate to the variation of the applied stresses, and high reliability can be realized by the low sensitivity of the laminate strengths as well as the strength itself. Although the biaxial laminates optimally designed for a deterministic loading give high strength, triaxial or tetra-axial laminates are better for a probabilistic condition.

The reliability of the multiaxial laminate becomes high with the increase in the number of orientation axes. This is much different from the results for the deterministic conditions, where the strength becomes low when the number of orientation axes increases. The comparison between the optimum fiber orientation angles under deterministic and probabilistic conditions is summarized in Fig. 3. Optimum fiber angles are much different betweeen the deterministic and probabilistic conditions and among the three types of multiaxial laminates. These optimum values generally increase when the applied stresses have some uncertainties. This means that the optimum laminates approach to quasi-isotropic plates under probabilistic conditions. This is clearly seen from the result for the triaxial laminate, where the optimum fiber orientation angles are around $\theta = 60$ deg under probabilistic loadings.

Simultaneous optimization of fiber orientation angles and ply ratios yields better configurations than those for constant ply ratios. The effects of different distributions for stresses and strengths on reliability are also investigated, and the reliability of the laminate changes very much with the different distributions, whereas the optimum fiber angle changes slightly. The effects of the correlations between applied stresses and strengths are also investigated.

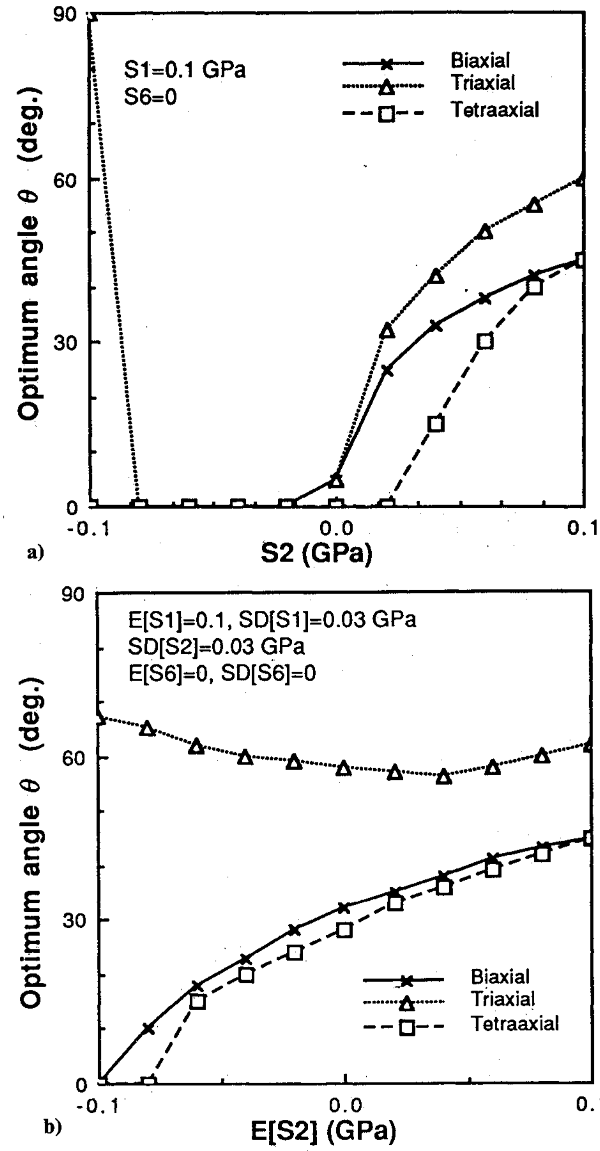

Fig. 3 Optimum orientation angles of multiaxial laminates: a) under deterministic conditions, and b) under probabilistic conditions.

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